Gödel's Koan

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Abstract

"A koan is a story, dialogue, question, or statement, which is used in Zen practice to provoke the "great doubt" and test a student's progress in Zen practice." (Wikipedia)

In 1968 William Howard proposed an assignment of ordinals $< \epsilon_0$ to terms for primitive recursive functionals of finite type with the property that the reduction of a term always lowers its ordinal measure. In a certain sense, it was Howard's assignment that motivated the apearence in 2014 of problem 26 (submitted by Henk Barendregt) in the TLCA list of open problems, the so-called Gödel's Koan.

Problem 26 - TLCA list of open problems
Submitted by Henk Barendregt
Date: 2014
Statement: Assign (in an 'easy' way) ordinals to terms of the simply typed lambda calculus such that reduction of the term yields a smaller ordinal.
Problem Origin: First posed by Kurt Gödel.
Construct an easy assignment of (possibly transfinite) ordinals to terms of the simply typed lambda calculus, i.e., a map

$F : \Lambda_{\rightarrow} \Rightarrow \{\alpha : \alpha \text{ is an ordinal}\}$ such that

$$\forall M, N \in \Lambda_{\rightarrow}\ [\mathrm{M} \rightarrow_{\beta} \mathrm{N} \Rightarrow \mathrm{F[N]} < \mathrm{F[N]}].$$

The aim of this paper is to explore three different routes to the solution of Gödel's Koan:

- Disatrous derivations
- Mimpgraphs
- Gentzen's reductions